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Abstract

In this paper a bank faces excess demand in the loan market, can sort loan applicants by an observable measure of quality, and faces a small but positive probability of default. The bank uses two policies to allocate credit: (i) tighten restrictions on loan quality; (ii) limit the number of loans of a given quality. We show that the level of default risk and other structural conditions have important effects on the market for loanable funds and the bank's optimal policies (loan rates, deposit rates, and lending standards). The structural conditions that we examine are monitoring costs, returns on alternative investments, firms' minimum funding requirements, and the level of the reserve requirement. The model provides insight into several stylized facts observed in loan markets, especially in developing countries.

JEL Classification Numbers: G10, G32

Keywords: Default Risk; Banks; Credit Rationing; Developing Countries; Interest Rate Spreads; Monitoring Costs

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1 Introduction

Banks are the dominant financial institution for channeling funds from savers to entrepreneurs in most "emerging financial markets." Many countries, especially developing economies, report the following problems (cf., Beim and Calomiris (2000)): costly banking crises; large spreads between deposit and loan rates; and reports of "credit crunches" (i.e., excess demand) in loan markets. We construct a model of a risky bank that can account for these stylized facts. The bank arises endogenously to accept deposits from investors and make loans to entrepreneurs with risky projects that can be sorted by an observable measure of project quality. The bank faces a small but positive probability of default. This friction in the bank's loan portfolio causes depositors to consider the risky bank's profitability. Specifically, depositors require a risky bank to be more profitable than a riskless bank because they must be compensated for the expected cost of recovering funds when default occurs.

We analyze the problem of a bank that chooses a deposit rate, loan rate, and loan portfolio quality when there is excess demand for loans and a reserve requirement. There is no deposit insurance. The bank manages the excess demand by rationing loans in two ways. First, because the bank chooses the quality of its loan portfolio, the bank can tighten the minimum quality requirement for loan applicants.[1] Second, the bank can restrict the quantity of loans it grants to borrowers of a given quality level. Rationing by loan quantity was proposed by Williamson (1986) for banks that are not subject to default risk. To our knowledge, rationing by loan quality has not been studied previously in equilibrium models,[2] yet an important role of banks is to screen loan applicants based on measures of project quality. We assume that the quality of individual projects is observable by the bank, and focus on the implication of loan portfolio risk for loan rates, deposit rates, and lending standards.

The level of default risk and other structural conditions have important effects on the market for loanable funds and a risky bank's equilibrium decisions. As one might expect, default risk is "priced out." We show that the default premium that this risk induces can affect the deposit rate, the loan rate, or quality cutoff, and gives rise to four distinct equilibrium outcomes:

[1]For example, quality can be measured by a parameter that indexes a mean preserving change in the variance of the distribution of a bank's loan portfolio returns.

[2]There is a literature on loan portfolio diversification, but it is aimed at operational ways to measure and control a bank's credit risk exposure. Our focus is on the implications of a given level of default risk for the macroeconomic problems enumerated at the outset.

(i) Rationing by loan quality: The default premium is borne entirely by the loan rate. Neither the extent of credit rationing nor the deposit rate change. The change in the interest rate spread is larger than the change in the default premium, a type of multiplier effect.

(ii) Rationing by loan quantity: When the bank's expected return for a given quality level is insufficient to compensate depositors, increases in the default premium increase rationing by loan quantity and decrease the deposit rate. The decrease in the deposit rate causes dis-intermediation.

(iii) Both types of rationing can occur if the default risk is sufficiently high.

(iv) No banking equilibrium: For some parameter configurations no banking equilibrium exists. This case corresponds to the costly banking crises observed worldwide.[3]

The paper analyzes each case and characterizes conditions under which the case occurs.

2 The Model

Consider a model with two types of risk neutral agents, α lenders and $1-\alpha$ entrepreneurs. There is an initial planning period, and a subsequent consumption/production period. Each entrepreneur is endowed with a project with a random return y_i but no input. Hence entrepreneurs wish to borrow. Each lender is endowed with one unit of input but no project. All projects have a common scale $q > 1$. Agents are asymmetrically informed. Borrowers privately and costlessly observe their return, but lenders do not unless a state verification cost is paid. If a lender chooses to incur cost $c_b > 0$ to verify the entrepreneur's project return, this cost is paid in output to an exogenous verification authority. Deadweight loss c_b "disappears" from the economy. The true project realization y_i is *privately* revealed only to the lender who pays the cost.

Williamson (1987) established that in this costly state verification model a bank emerges endogenously from among the set of investors. The bank writes

[3] The IMF estimates that the cummulative output loss due to banking crises as a percentage of GDP is 10.2 % among industrial countries and 12.1 % among developing countries (cf., IMF (1998), Table 15, p. 79). Our results suggest that differences in the default premium and structural differences may account for some of this.

deposit contracts with investors and loan contracts with entrepreneurs. To this standard setting where individual project returns are identically and independently distributed and described by the common distribution function $G(y_i)$, we add two features that affect the distribution of average returns from the bank's loan portfolio $G(y, \theta; s)$. First, we introduce two states, $s = l, h$, where the bank defaults in the low state and is solvent in the high state. Second, we introduce an index θ that measures project quality.

Assume that $G(y, \theta; s)$ is defined over the range of possible returns $(0, \bar{y})$ and has density function $g(y, \theta; s)$.

- As θ changes, $G(y, \theta; s)$ changes with $G_\theta(y, \theta) \geq 0$ for all y and $G_\theta(y, \theta; s) > 0$ for some y.[4] This specification, for example, captures the situation of a bank that faces a distribution of entrepreneur projects with returns that have the same mean but different variances. Quality parameter θ has a distribution $H(\theta)$ over a range $\theta_{min}, \theta_{max}$ with density $h(\theta)$.
- States $s = l, h$ affect distribution $G(y, \theta; s)$ in the sense of Second Order Stochastic Dominance: $G_l(y, \theta; s) \geq G_h(y, \theta; s) > 0$ for all y.

Assume that θ and s are independent. Let $\bar{s}$ denote no default risk and p_s be the probability of state s. Assume that s does not affect the expected return

$$p_l \int_0^{\bar{y}} y dG(y, \theta; s = l) + p_h \int_0^{\bar{y}} y dG(y, \theta; s = h) = \int_0^{\bar{y}} y dG(y, \theta; \bar{s}) = \tilde{y}.$$

Lenders, who have an endowment of input but no project, inelastically supply labor when young to earn wage $w > 0$, and have access to two investment opportunities. First, they may lend to entrepreneurs under terms governed by a contract. Second, they may invest in an outside option that yields $x_i > 0$ for each unit invested. Return x is costlessly observable and does not require verification.[5] Prior

[4] As θ increases the distribution is more risky in the sense of Second Order Stochastic Dominance. When agents are risk neutral, a mean-variance selection rule is appropriate for a normal distribution of returns, cf., Bawa (1975). In general, an increase in the variance of the distribution of loan returns decreases the "quality" of loan applicants, decreasing the bank's expected return (cf., Section 6).

[5] This introduces an upward sloping supply curve for saving deposits. The outside option can be motivated as a government bond with a publicly known return. In contrast, the returns on private projects are costly to verify.

to its realization, x is uncertain and has a distribution $I(x)$, with $i(x) = I'(x) > 0$ and $x \epsilon\ 0, \bar{x}$, where $\bar{x}$ is the maximum return on the outside opportunity.

Finally, information is crucial in the economy. We assume that

- Ex-ante agents know $G(y, \theta; s)$, $I(x)$, $H(\theta)$, θ, p_l
- Ex-post entrepreneurs privately observe return y_i, and investors do not unless costly verification occurs. Return x is costlessly observed by all.

2.1 Distribution of Bank's Portfolio

We now derive the relationship among y, θ, s, and the probability of default, p_l. We begin by distinguishing between the bank's income from an individual borrower and the average income from its loan portfolio. The bank's income from entrepreneur $i = 1, \ldots, m$ is

$$L_i(x_i) = L_i(G(y_i, \theta_i; s)).$$

The average income per borrower from the loan portfolio under contract $L(.)$ is

$$\frac{1}{m}\sum_{i=1}^{m} L_i(G(y_i, \theta_i; s)) \rightarrow EL(G(y, \theta; s)|s).$$

$G(\cdot)$ is the distribution of returns from the bank's loan portfolio $L(G(y, \theta; s)|s)$. Assume that $G(\cdot)$ takes two values given by

$G_l(\cdot)$: The distribution of returns from the bank's loan portfolio if $s = l$

$G_h(\cdot)$: The distribution of returns from the bank's loan portfolio if $s = h$

Krasa and Villamil (1992, p. 203) shows that the probability of bank failure, p_l, converges to the probability that the return from the bank's asset portfolio is less than return that the bank must pay the depositors, face value $\bar{D}$

$$P(\{\frac{1}{m}\sum_{i=1}^{m} L_i(G(y_i, \theta_i; s)) < \bar{D}\}) \rightarrow P(\{EL(G(y, \theta; s)|s) < \bar{D}\}).$$

We assume that the bank defaults in the low state, with $p_l > 0$ but small

$$P(\{\frac{1}{m}\sum_{i=1}^{m} L_i(G(y_i, \theta_i; s = l)) < \bar{D}\}) = p_l.$$

2.2 Riskless Banking

When the bank faces no default risk, Williamson (1986) showed that (i) the optimal contract is simple debt, (ii) banks arise endogenously to eliminate duplicative monitoring, and (iii) equilibrium credit rationing by loan quantity may arise. We briefly review these results in our model in Appendix 1 since they provide a benchmark to which the bank's problem with default risk will be compared. The Appendix shows that when the bank faces no default risk, i.e., $s = \bar{s}$, the expected return function for a bank that contracts with an infinite number of entrepreneurs is

$$\Pi(L(y,\theta),\theta;\bar{s}) = \int_{B_b}(L(y,\theta) - \frac{c_b}{q})dG(y,\theta;\bar{s}) + \int_{B'_b}\bar{L}dG(y,\theta;\bar{s}). \quad (1)$$

The first term on the right hand side gives the bank's expected return from loan contract $L(y,\theta)$, net of per project monitoring costs, $\frac{c_b}{q}$, in default states $y \in B_b$. The second term gives the bank's expected return when loans are fully repaid at face value $\bar{L}$ in non-default states $y \in B'_b$.

In a perfectly competitive market, a riskless bank equates the expected return function with the interest rate on deposits, $\bar{D}$. Williamson showed that the depositors' expected cost of monitoring the bank goes to zero as portfolio size goes to infinity because the portfolio earns $\bar{L}$ with probability one. The bank can then pay depositors reservation value $\bar{D}$ with certainty. The bank never defaults and the cost of delegation is nil. Williamson also showed that the bank's expected return function $\Pi(\cdot)$ is concave in loan rate $\bar{L}$, thus it has an interior maximum at some $\bar{L}^*$. This can lead to equilibrium credit rationing by loan quantity at $\bar{L}^*$. Even if a rationed borrower offered to pay a loan rate higher than $\bar{L}^*$, the bank would refuse because $\bar{L}^*$ maximizes the bank's expected return.[6]

2.3 Risky Banking

When a bank may default in some states, Krasa and Villamil (1992) showed the following. (i) Banking remains optimal if monitoring costs are bounded and the default probability for the loan portfolio is sufficiently small. (ii) The optimal contract is two-sided debt, where $(L(y,\theta), B_b)$ is the loan contract between the bank and entrepreneurs, and $(D(y), B_d)$ is the deposit contract between the bank and

[6]The intuition for this credit rationing by loan quantity is that when failure is costly to the lender, an increase in the loan rate may decrease the bank's expected return because it raises the probability of borrower default.

lenders. As before, the bank funds m entrepreneurs using deposits from $mq - 1$ lenders. However, on set B_b some projects default and the bank incurs monitoring cost c_b, and on B_d the bank defaults and the $mq - 1$ depositors incur monitoring cost c_d.

When banking is risky and default occurs in state $s = l$, the bank's incentive constraint, which insures that it requests costly state verification of entrepreneurs in bankruptcy states, depends on:

(i) Bank assets: revenue from loan portfolio $\pi(\cdot) = q \sum_{i=1}^{m} min(L(y_i, \theta_i), \bar{L}_i)$

(ii) Bank liabilities: the bank owes depositors $D(\pi_b(L, \theta; s))$

(iii) Bank costs to monitor y_i that default: $C = c_b N(s)$

A risky bank's ability to repay depositors (i.e., its liabilities) depends on its asset portfolio. Assume that the bank's total revenue is homogeneous. Then the bank's incentive constraint is

$$\sum_{s=l,h} p_s \left[\pi(L, \theta; s) - D(\pi_b(L, \theta; s)) - C(s)\right] = \frac{\bar{D}}{q}. \tag{2}$$

Because the bank arises endogenously (i.e., investors delegate monitoring to one investor), the bank must earn the same expected return per project as the remaining investors, $\bar{D}/q$.[7]

The depositor's incentive constraint, which insures that depositors request costly state verification of the bank in bankruptcy states, is derived as follows. Depositors must monitor whenever $D(\pi_b(L, \theta; s))$ is less than $\bar{D}$, incurring cost $C_d = c_d M(s)(mq - 1)$, where $M(s)$ is a binary variable that equals one if the bank defaults and the $mq - 1$ depositors monitor and zero otherwise. Thus, the depositor's incentive constraint is given by

$$\sum_{s=l,h} p_s \left[D(\pi_b(L, \theta; s)) - C_d(s)\right] = \frac{\bar{D}}{q}(mq - 1). \tag{3}$$

As the number of loans goes to infinity, the bank can eliminate idiosyncratic risk but not default risk. Thus, the income from its loan portfolio may not be sufficient

[7] See Williamson (1986) or Krasa and Villamil (1992) for proofs of the optimality of delegated monitoring relative to direct investment without and with risk, respectively.

to fully repay depositors in some states. In those states the depositors must monitor the bank. We assume that the bank defaults in state $s = l$. The risky bank's expected return function, which must be non-negative, is

$$\sum_{s=l,h} p_s \left[\int_{B_b} (L(\cdot) - \frac{c_b}{q}) dG(\cdot) + \int_{B_b'} \bar{L} dG(\cdot) - D(\pi_b(L, \theta; s)) \right]. \quad (4)$$

2.4 Comparison of Riskless vs. Risky Banking

In Section 2.2, (1) established that

$$\Pi(\cdot) = \bar{D}.$$

In Appendix 2 we show that because a risky bank will sometimes default, the expected monitoring costs that depositors incur raise the effective reservation return to

$$\Pi(\cdot) = \bar{D} + \rho.$$

The term $\rho = p_l q c^d$ reflects the cost of default. This risk premium depends on the size of depositor monitoring cost c^d, the project scale q, and the probability that the low state will occur, p_l. The bank's expected return function $\Pi(L(y, \theta), \theta; s)$, given by the left-hand-side of (15) in Appendix 2, has two important properties. See the Appendix for the proof.

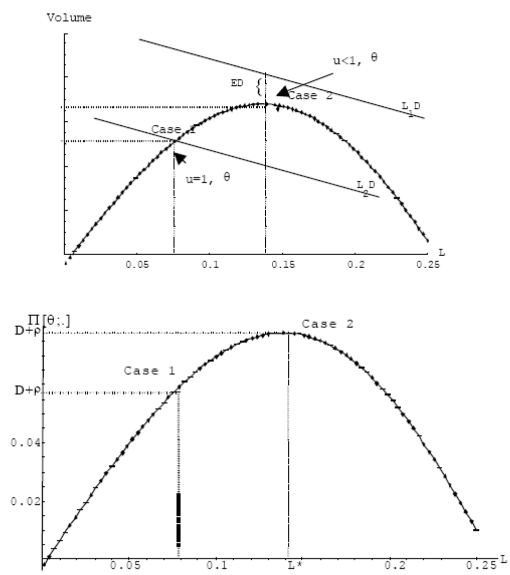
Proposition 1: Assume $c_b g(0, \theta) < q$ and $\frac{c_b}{q} g_x(x, \theta) + g(x, \theta) > 0$.[8]
(a) $\Pi(L(y, \theta), \theta; s)$ is concave in L, given θ.
(b) $\Pi(L(y, \theta), \theta; s)$ is decreasing in θ, for $\bar{L} = \bar{L}^*$ and given $\bar{D}$.

Property (a) is Williamson's credit rationing result for a fixed portfolio quality level, θ. Williamson (1986) showed that in the costly state verification model with no risk of bank default, credit rationing by loan quantity can occur because the bank's expected return function is concave. Concavity follows from the fact that an increase in the loan rate has two effects on $\Pi(\cdot)$: Revenue increases as $\bar{L}$ increases, but expected monitoring costs also increase. The second effect occurs because raising $\bar{L}$ raises the probability that bankruptcy will occur. The second effect may dominate the first for sufficiently high loan rates. Concavity implies that

[8]These assumptions are standard. For example, see Boyd and Smith (1997).

there is an optimal loan value $\bar{L}^*$. When credit rationing by loan quantity occurs, some borrowers are fully funded while other observationally identical borrowers are not.[9] A rationed entrepreneur will not get additional credit even if the agent is willing to pay $\bar{L} > \bar{L}^*$ because this would reduce bank profit. See Figure 1.

FIGURE 1. Credit Rationing



Property (b) states that the expected return function is decreasing in θ. To fix ideas, suppose that quality is measured as a mean preserving change in the variance of the distribution of loan returns, where an increase in θ decreases the "quality" of loan applicants. The Figure shows that as θ increases, with $\theta^B > \theta^A$, the expected return function $\Pi(\cdot))$ shifts down.

Proposition 2 establishes that there is an optimal quality cutoff level, θ^A. The proof is in Appendix 2.

Proposition 2: Assume $c_b g(0, \theta) < q$ and $\frac{c_b}{q} g_x(x, \theta) + g(x, \theta) > 0$. Then there is an optimal quality threshold level θ^A such that

[9] Credit rationing by loan quantity operates as follows. Suppose that loan demand is $(1 - \alpha)q$ and loan supply is α with $w = 1$. If at $\bar{L}^*$ there is excess demand in the loan market, then $(1 - \alpha)q > \alpha$. In order to ration this excess demand α borrowers are randomly selected from the $(1 - \alpha)q$ potential borrowers. Those α borrowers are fully funded at q units each. The other observationally identical borrowers get zero.

- If $\theta_i \leq \theta^A$: the entrepreneur is financed
- If $\theta_i > \theta^A$: the entrepreneur is rationed

Proposition 2 indicates that banks sort loan applicants based on quality using critical value θ^A. All θ above the threshold (i.e., high variance or low quality applicants) are rationed. Threshold quality level θ^A is an additional form of credit rationing that to our knowledge has not been considered in the economic literature previously. In the remainder of the paper we analyze the effect of default risk on these two forms of credit rationing, loan quantity and loan quality.

3 The Loan Market

Equilibrium in the loan market results from the equality of demand by borrowers and supply by lenders. Each borrower demands q units of credit to invest in the fixed scale project. Total loan demand is thus $(1-\alpha)q$. Propositions 1 and 2 show that credit rationing can occur for two distinct reasons, thus we model the loan market as follows. Let $u \leq 1$ be the fraction of entrepreneurs that receive credit for a given quality level θ^A. Proposition 1 shows that credit rationing by loan quantity, $u < 1$, is due to the concavity of the bank's expected profit function. Proposition 2 shows that banks also ration credit by adjusting quality cutoff θ^A. Since $H(\theta^A)$ is the distribution of project quality, by varying θ^A the bank adjusts portfolio quality to clear the market.

The demand for bank loans by entrepreneurs is $(1-\alpha)quH(\theta)$. The total supply of funds is αw. Because the α lenders have an outside investment opportunity with return x, they will divert funds away from banks if x exceeds the deposit interest rate $\bar{D}$. Then the supply of funds by depositors to banks is $\alpha w H(\bar{D})$. Assume that the economy has excess credit demand, $(1-\alpha)q > \alpha w$. Then the loan market equilibrium is given by

$$(1-\alpha)uqH(\theta) \geq \alpha w H(\bar{D}).$$

Finally, banks must satisfy a reserve requirement $\bar{\delta}$ that constrains the amount the bank can lend. A reserve requirement has two effects

(i) Banks face an additional constraint, $\delta(\theta) \geq \bar{\delta}$, where $\delta(\theta) = (1 - H(\theta)) - k$. This specification of $\delta(\theta)$ captures the idea that banks choose the optimal θ^A given the reserve requirement. Constant k takes into account that banks choose portfolio quality even if $\bar{\delta}$ is zero.

(ii) Banks must keep a proportion of deposits on hand to satisfy the reserve requirement. This further reduces the supply of credit to

$$(1-\alpha)uqH(\theta) \geq \alpha w H(\bar{D})(1-\bar{\delta}).$$

4 Credit Rationing

Assume perfect competition. We now state the bank's problem, and analyze it with and without default risk. Let $\rho = p_l q c^d$ denote default premium. When $\rho = 0$ there is no default risk and when $\rho > 0$ default risk exists.

The Bank's Problem. Choose $\bar{L}$, $\bar{D}$, and θ to maximize

$$\Pi(L,\theta) = \geq \bar{D} + \rho, \tag{5}$$

subject to:

$$(1-\alpha)uqH(\theta) \leq \alpha w H(\bar{D})(1-\bar{\delta}), \tag{6}$$

$$(1-H(\theta)) - k \geq \bar{\delta}. \tag{7}$$

The bank chooses loan and deposit rates and a portfolio quality threshold to maximize its expected return.[10] Depositor incentive compatibility, (5), requires a risky bank's expected return to be at least as great as the risk augmented depositor reservation level, $\bar{D}+\rho$. The bank is also constrained by loan market equilibrium condition (6), which acts as a feasibility constraint, and the reserve requirement (7).

Our goals are two-fold. First, we analyze the factors that affect the two types of credit rationing. We pay particular attention to portfolio quality selection (i.e., the bank's choice of threshold θ^A) since portfolio quality selection is a core operational function of a bank, it is intrinsically related to default risk, and the factors that affect θ^A have not been studied previously. Portfolio quality selection is irrelevant for a riskless bank, but it is crucial for "risky banks." Second, we will show

[10] Appendix 2 shows that after integration by parts, (5) is

$$\Pi(L,\theta) = L - \frac{c_b}{q} G(L,\theta;s) - \int_0^L dG(y,\theta;s).$$

both analytically and quantitatively that default risk interacts with both types of credit rationing to deepen the distortions in loan markets. The results that we derive help explain the costly banking crises, large interest rate spreads, and "credit crunches" observed in many developing countries.

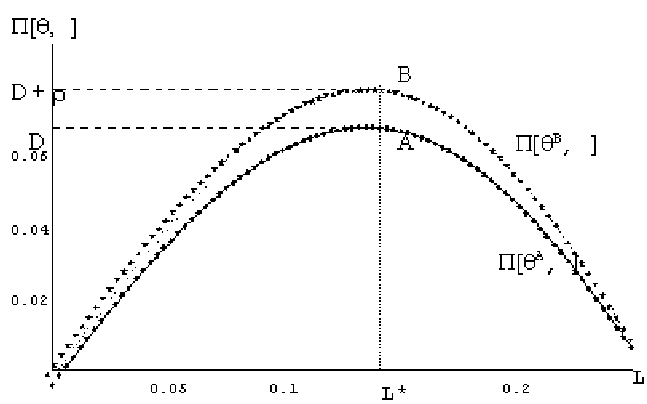
To solve the bank's problem, we consider two cases described by Figure 2:

- Rationing by loan quantity, $u < 1$: Not all borrowers of a given quality who request a loan receive one. $\bar{L}$ is fixed at the maximum income level for a given θ, $\bar{L}^*(\theta)$.[11] Banks choose $\bar{D}$, θ and indirectly u.

- Rationing by portfolio quality, $u = 1$: Since $\bar{L}$ is fixed, banks maximize with respect to $\bar{L}$, $\bar{D}$ and θ.

To simplify the analysis, assume that the distribution of returns on the outside alternative is uniform, thus $I(x) = \frac{x}{\bar{x}}$, where $x = \bar{D}$ in a competitive market.

FIGURE 2. Default risk and quality selection



The Figure illustrates that when there is default risk, the loan rate must be higher than when there is no default risk. This has implications for quality selection. Compared to a situation with no default risk, and given the fact that the expected return function is decreasing in θ, for the same loan rate a risky bank is

[11] The condition for this type of credit rationing is given in Proposition 3 below.

stricter about entrepreneur quality. The expected return function evaluated at the optimal loan rate $\bar{L}^*(\theta)$ is:

$$\Pi(\bar{L}^*(\theta), \theta) = \psi(\theta).$$

Proposition 1 shows that this function is decreasing in θ.[12] Proposition 2 shows that for a given $\bar{D}$, there is an optimal θ^A. Depositors at a risky bank must be compensated for the expected monitoring costs they bear. Then for the same loan rate, bank profit must be larger relative to a riskless bank. As a consequence, riskier banks tighten their optimal θ. Thus $\theta = \theta^C$ is lower than θ^A, meaning that banks are more selective about quality. Ceteris paribus, default risk increases quality rationing: Entrepreneurs with qualities between $\theta^A > \theta > \theta^C$ are rationed now.

4.1 Credit Rationing by Loan Quantity: $u < 1$

When credit rationing by loan quantity occurs, the fraction of entrepreneurs of a given quality that receive loans is less than one (i.e., $u < 1$) and $\bar{L}$ is fixed at the maximum income level for a given θ, $\bar{L} = \bar{L}^*(\theta)$. Banks choose $\bar{D}$, θ and indirectly u (i.e., the fraction of loan requests to grant). Consider the equations in the bank's problem, (5), (6) and (7).

Bank expected return is $\Pi(\bar{L}^*(\theta), \theta) = \psi(\theta)$. From (5), for a riskless bank $\Pi(\bar{L}^*(\theta), \theta) = \bar{D}$, and for a risky bank $\Pi(\bar{L}^*(\theta), \theta) = \bar{D} + \rho$. Since $u < 1$, loan market equilibrium condition (6) is

$$u = \frac{\alpha w(1-\bar{\delta})\bar{D}}{(1-\alpha)qG(\theta^A)\bar{x}} < 1. \tag{8}$$

There is no change in the reserve requirement (7).

A riskless bank's expected revenue equals the deposit rate, $\psi(\theta) = \bar{D}$. Solving (8) for $\bar{D}$ and imposing $\psi(\theta) = \bar{D}$ gives

$$\psi(\theta) < \frac{(1-\alpha)qG(\theta^A)\bar{x}}{\alpha w(1-\bar{\delta})}. \tag{9}$$

This condition means that the bank cannot obtain sufficient expected return from its loan portfolio to pay depositors the market clearing deposit rate. As a consequence, rationing by loan quantity arises (see Case 1 in Figure 3); the bank can

[12] The interest rate on loans is endogenous, and depends on the distribution of project returns. It can decrease, increase or remain constant when θ changes. We assume that it remains constant.

not finance all applicants.[13]This situation arises whenever the return from borrowers is not sufficient to cover the deposit rate. Williamson showed that this type of rationing can occur for riskless banks, but we will now show that default risk deepens the problem.

When there is default risk and $u < 1$, $\bar{L}$ is fixed at the maximum income level for a given θ. Comparative static results in Claim 4.11 show the following. First, default risk does not affect the quality cutoff, θ^A. Second, quantity rationing increases as default risk increases. Third, the deposit rate goes down by the same amount as the increase in the default premium, due to a decrease in u (i.e., an increase in credit rationing by loan quantity). Thus, the deposit rate and u adjust to equilibrate the bank's expected revenue and the deposit rate.

Claim 4.11. When banks ration credit by loan quantity (i.e., $\bar{L}(\theta) = \bar{L}^*(\theta)$, (9) is satisfied and $\rho > 0$), then as the default premium increases

(i) $\frac{d\theta}{d\rho} = 0$: There is no effect on portfolio quality.

(ii) $\frac{du}{d\rho} = -\frac{\alpha w(1-\bar{\delta})}{\bar{x}(1-\alpha)qH(\theta)} < 0$: Quantity rationing increases (u declines).

(iii) $\frac{d\bar{D}}{d\rho} = -1$: There is a one-for-one decrease in the deposit rate.

To understand the intuition for Claim 4.11, recall that $\bar{D} + \rho = \psi(\theta)$. Thus, if it were the case that $u = 1$, then (9) implies that for a risky bank

$$\psi(\theta) - \rho < \frac{(1-\alpha)qH(\theta)\bar{x}}{\alpha w(1-\bar{\delta})}. \tag{10}$$

This equation means that if the risky bank granted all loan requests at the given quality level ($u = 1$), it would not obtain an expected return sufficient to pay the market clearing rate on deposits. Thus the bank cannot finance all applicants, because default risk causes the expected return on the bank's portfolio to decrease. Equation (10) shows that this credit rationing by loan size is more likely to occur as default risk increases. We summarize this result formally in Proposition 3.

Proposition 3: When $\bar{L} = \bar{L}^*$, (10) is satisfied, and $\rho > 0$, then credit rationing by loan size occurs, i.e., $u < 1$.

Proposition 3 establishes that in order for quantity rationing to occur, banks are already at the maximum expected revenue and $\bar{L}(\theta) = \bar{L}^*(\theta)$. Therefore an

[13]Due to the minimum finance constraint, $q > 1$, it is not possible to give all borrowers "a haircut" (i.e., ration by size as in Stiglitz and Weiss (1984)). Rather, some borrowers of a given quality level are fully funded, and other observationally identical borrowers are completely rationed.

increase in default risk has no effect on the loan rate.[14] As the default premium (ρ) increases, banks become less profitable and attract less deposits. The outside opportunity becomes more attractive and banks lose their deposit base. As a result of this dis-intermediation, Claim 4.11 shows that quantity rationing increases since less funding is available for borrowers and the interest rate spread increases. The increase in risk has no effect on the quality cutoff in this case.[15]

Williamson showed that credit rationing by loan quantity can arise even when banks are not risky (i.e., $\rho = 0$). Proposition 3 indicates that default risks deepens this type of rationing because (10) is more likely to be satisfied when $\rho > 0$. To illustrate this, we perform comparative statics on (10). Assume that equation (10) holds as an equality. Then:

Claim 4.12. As w, α, $\bar{x}$, $\bar{\delta}$, or q increase,
(i) $\frac{d(\psi(\theta)-\rho)}{dw} = -\frac{(1-\alpha)qH(\theta)\bar{x}}{\alpha w^2(1-\bar{\delta})} < 0$ and $\frac{d(\psi(\theta)-\rho)}{d\alpha} = -\frac{qH(\theta)\bar{x}}{\alpha^2 w(1-\bar{\delta})^2} < 0$
(ii) $\frac{d(\psi(\theta)-\rho)}{d\bar{x}} = \frac{(1-\alpha)qH(\theta)}{\alpha w(1-\bar{\delta})} > 0$; $\frac{d(\psi(\theta)-\rho)}{d\bar{\delta}} = \frac{(1-\alpha)qH(\theta)\bar{x}}{\alpha w(1-\bar{\delta})^2} > 0$; and $\frac{d(\psi(\theta)-\rho)}{dq} = \frac{(1-\alpha)H(\theta)\bar{x}}{\alpha w(1-\bar{\delta})} > 0$

Parts (i) indicates that credit rationing by loan quality is less likely if there is an increase in the supply of funds, due to either an increase in wages or an increase in lenders. Part (ii) indicates that credit rationing is more likely in two cases. First, if there is a decrease in the supply of funds, due to an increase in the return on the outside opportunity or reserve requirement. Second, if there is an increase in the demand for funds due to an increase in the minimum project scale.

4.2 Credit Rationing by Loan Quality: $u = 1$

Assume that there is no credit rationing by loan quantity, so $u = 1$. Banks choose $\bar{L}$, $\bar{D}$ and θ.[16] Given the reserve requirement, banks select θ so that,

$$\delta(\theta^A, 1) = \bar{\delta}.$$

[14] In fact, an increase in ρ can trigger the transition from only quality to quality and quantity rationing. Table 7 for a numerical example which shows that for a riskless bank $u = 1$, but when default risk ρ increases then $u < 1$ and rationing by loan quantity occurs.

[15] Note that (7) fixes the cutoff θ^A. The bank attains the maximum expected revenue, but the supply of loanable funds is insufficient to finance all loan applicants (i.e., clear (6)). See Guzman (2001), footnote 6.

[16] Given the assumption that $H(\bar{D})$ has a uniform distribution and that $H(\bar{D}) < 1$, then $(1-\alpha)q > \alpha w H(\bar{D})$ (there is excess demand in the loan market for projects).

Then solving equations (6) and (7) with $u = 1$, we get $\bar{L}_A$ and $\bar{D}$ such that

$$(1-\alpha)qG(\theta^A) = \alpha w H(\bar{D})(1-\delta(\theta^A, 1)).$$

First consider the case for a riskless bank. The interest rate on deposits equals

$$\bar{D} = \frac{(1-\alpha)qG(\theta^A)\bar{x}}{\alpha w(1-\bar{\delta})}. \tag{11}$$

The interest rate on loans is the $\bar{L}_A$ that solves

$$\pi(\bar{L}_A, \theta^A) = \frac{(1-\alpha)qG(\theta^A)\bar{x}}{\alpha w(1-\bar{\delta})}.$$

This interest rate is lower than $\bar{L}^* = \eta(\theta^A)$ since there is no rationing by loan quantity ($u = 1$). Now consider the case for a risky bank. Equation (5) holds and depositors must be compensated for default risk $\rho = p_l q c_m^d$. The bank's expected revenue function is now given by

$$\pi(\bar{L}, \theta) \geq \bar{D} + p_l q c_m^d.$$

To make the results comparable, we assume that the deposit rate is the same as in the case with no default risk. Banks again maximize expected revenue subject to equations (6) and (7), selecting $\bar{D}$, $\bar{L}$ and θ and taking into account the default premium. From (7), $\theta = \theta^A$ and $\delta(\theta^A, 1) = \bar{\delta}$. Equation (10) holds with equality, and $\bar{D}$ is the same as in the case with no default risk. Since the bank must now compensate depositors for the expected recovery cost in case of bankruptcy, the interest rate on loans is higher than when there is no default risk. Then results $\bar{L}_L > \bar{L}_A$. But this interest rate is still lower than $\bar{L}^* = \eta(\theta^A)$, since there is no rationing by loan quantity.[17]

Total differentiation of (5), (6) and (7) allows us to establish the following comparative static results about the effect of default premium on interest rates and the quality cutoff:

Claim 4.2. When banks ration credit by loan quality, then as the default premium increases

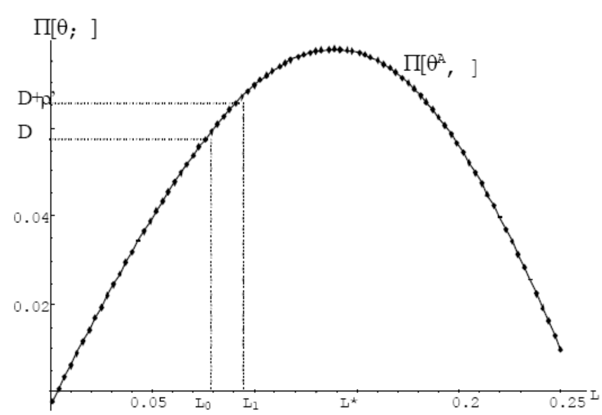
[17] As in Guzman (2000), we divide the analysis of the bank's problem into two cases: $u < 1$ and $u = 1$. As Proposition 3 indicates, for $u < 1$ to hold it must be the case that the bank is already at the maximum expected return level with $\bar{L} = \bar{L}^*(\theta)$. Otherwise, $u = 1$ and there is no rationing by loan quantity.

(i) $\frac{d\bar{L}}{d\rho} = \frac{1}{\pi_{\bar{L}}} > 0$: The loan rate increases.
(ii) $\frac{d\theta}{d\rho} = 0$: There is no effect on portfolio quality.
(iii) $\frac{d\bar{D}}{d\rho} = 0$: There is no effect on the deposit rate.

Under quality rationing an increase in default premium generates an increase in the loan rate. However, there is no effect on the quality cutoff or on the deposit rate. Only the spread is affected. Since banks have not reached the maximum expected return, an increase in the loan rate can still increase expected return. Therefore banks transfer the increase in the default premium to borrowers by increasing the loan rate. It is not necessary for banks to tighten quality.

In summary, Figure 3 shows that a default premium generates an increase in the loan rate from L_0 to L_1 due to the additional risk that banks must compensate depositors for (i.e., $\bar{D}+\rho > \bar{D}$). In order to do this, banks charge a higher rate on loans. This increases the observed spread between deposit and loan rates, a fact observed in many developing countries. Note that in Figure 3, $L_0 < L_1 < \bar{L}^*$. Since $L < \bar{L}^*$, the conditions of Proposition 3 are not satisfied and rationing by loan quantity does not occur.

FIGURE 3: Case 1 Credit Rationing



5 Conclusions

This paper analyzes the interaction between default risk and credit rationing in an economy with asymmetric information and costly monitoring. When portfolio default risk cannot be completely eliminated, it has an interesting impact on the bank's equilibrium decisions. We show that the size of the default premium affects the loan rate, the deposit rate, and hence the interest rate spread. The size of the risk, along with other parameters of the model, also affects which of four possible equilibria occur: (i) credit rationing by loan quality, (ii) credit rationing by loan quantity, (iii) both types of credit rationing, (iv) no banking equilibrium.

We obtain the following results. First, the model shows that under quality rationing only, the effect of default risk is borne entirely by the loan rate. Neither the extent of credit rationing nor the deposit rate changes, but the loan-deposit rate spread increases due to the default premium. Claim 4.2 shows that the change in the spread is larger than the change in the default premium, a type of multiplier effect. Second, we show that credit rationing by loan quantity can also arise when the bank's expected return for a given quality level is not high enough. This can happen if default premium is high and/or the distribution of returns is unfavorable. Claim 4.11 shows that under this type of credit rationing, the interest rate on loans is fixed at the maximum return. An increase in the default premium is reflected in an increase in rationing by loan quantity and a decrease in the deposit rate. Therefore, disintermediation results. Third, both types of credit rationing can occur when the default premium is sufficiently high.

These results are consistent with the stylized facts observed in many developing countries: large interest rate spreads, costly banking crises, and reports of "credit crunches." The model suggests that these problems could be reduced in two ways: First, by reducing the level of default risk. This could be accomplished by better portfolio diversification or insurance opportunities. Second, by improving structural conditions. This could be accomplished by reducing monitoring costs and lowering returns on outside opportunities such as government bonds. However, we believe that it is unlikely that portfolio risk can be eliminated completely. Numerical simulations show that even small amounts of default risk can have big effects. This model seems especially appropriate for developing economies where default premium is often an important factor.

References

[1] Boyd, J. H. and Smith, B. D. (1997). Capital market imperfections, international credit markets, and nonconvergence. Journal of Economic Theory 73, 335-364.

[2] Boyd, J. H. and Smith, B. D. (1998). Capital market imperfections in a monetary growth model. Economic Theory 12, 519-560.

[3] Boyd, J. H. and Smith, B. D. (1999). The use of debt and equity in optimal financial contracts. Journal of Financial Intermediation 8, 270-316.

[4] Diamond, D. W. (1984) Financial intermediation and delegated monitoring. Review of Economic Studies, 51, 303-414.

[5] Gale D. and Hellwig, M. (1985). Incentive-compatible debt contracts: The one period problem. Review of Economic Studies 52, 647-663.

[6] Gilbert, R. A., Meyer, A. P. and Vaughan, M. D. (1999). The role of supervisory screens and econometric models in off-site surveillance. Review Federal Reserve Bank of St. Louis November/December, pp 31-56.

[7] Gilbert, R. A., Meyer, A. P. and Vaughan, M. D. (2001). How healthy is the banking system?. The Regional Economist Federal Reserve Bank of St. Louis April.

[8] Guzman, M. G. (1999). Bank structure, capital accumulation and growth: A simple macroeconomic model. Working Paper. Federal Reserve of Dallas August.

[9] Hamada, K. and Sakuragawa M. (2000). Asymmetric information, banking policy and international capital movements. Mimeo.

[10] Jafee, D., Stiglitz, J. (1990). Credit Rationing. Handbook of Monetary Economics Vol II, 16. Edited by Friedman B. M and Hahn F. H.

[11] Keaton, W. R. (1979). Equilibrium credit rationing. Garland Publishing, Inc. New York & London, 838-885.

[12] Krasa, S. and Villamil, A. (1992) A theory of optimal bank size. Oxford Economic Papers 44, 725-749.

[13] Rajan, R. (1992). Insiders and outsiders: the choice between informed and arm's length debt. Journal of Finance. XLVIII, 4, pp 1367-1400.

[14] Williamson, S. D. (1986) Costly monitoring, financial intermediation, and equilibrium credit rationing. Journal of Monetary Economic 18, 159-179.

[15] Williamson, S. D. (1987) Costly monitoring, loan contracts, and equilibrium credit rationing. The Quarterly Journal of Economics. February, 135-145.

[16] Williamson, S. D. (1987) Financial intermediation, business failures, and real business cycles. Journal of Political Economy. vol 95, 6, 1196-1216.

[17] Winton, A. (1995) Delegated monitoring and bank structure in a finite economy. Journal of Financial Intermediation. vol 4, 158-187.

Appendix 1

Williamson (1986) considered the following problem. Entrepreneurs propose loan contracts in a planning period that are analyzed by a lender. A contract is a pair $(L(y, \theta), B_b)$, where $L(y, \theta)$ is the loan repayment from an entrepreneur and B_b is the set of realizations where the entrepreneur is monitored. Given the feasible set of returns $0, \bar{y}$, costly state verification occurs on set B_b. No monitoring occurs on the complement $B_b' = 0, \bar{y} - B_b$.

Simple debt is optimal because it minimizes expected monitoring costs. Given realization y, the entrepreneur repays a fixed amount $\bar{L}$ which is not contingent on y, if $y \in B_b'$. If $y \in B_b$, the entrepreneur transfers the entire y to the bank. Incentive compatibility requires a fixed loan repayment $\bar{L} > 0$ in states where no costly state verification occurs. This fixed amount is given by $\bar{L} \leq argmin_{y \in B_b} y$. Williamson showed that the entrepreneur has the incentive to repay $\bar{L}$ when this is feasible because it economizes on deadweight monitoring costs. The entrepreneur keeps the difference, $y - \bar{L}$, as profit. For low realizations $y \in B_b$, the bank monitors, the entrepreneur gets zero, and the bank recovers $y - c_b$. Then $\bar{L}(y, \theta) \leq y$, $\forall y \in B_b$. Given this condition, $B_b = 0, \bar{L})$, since for any $y \geq \bar{L}$ the entrepreneur prefers to pay $\bar{L}$.

First, Williamson showed that simple debt contract $\bar{L}$ is optimal relative to any other alternative debt contract A because it minimizes expected monitoring costs.[18] Consider two optimal contracts $\bar{L}$ and A. To give the borrower the same expected return, the face value of A must be strictly higher: $\bar{A} > \bar{L}$. Then clearly the expected monitoring costs are less for contract $\bar{L}$ (i.e., the bankruptcy set where costly monitoring occurs $B_b^L \subset B_b^A$).

Second, Williamson showed that banking (i.e., delegated monitoring) is optimal because it eliminates costly duplicative monitoring. If a bank contracts with m entrepreneurs, then loan demand is mq since q is the scale of each project. In order to satisfy this demand the bank needs $mq - 1$ lenders. The bank receives $L(y, \theta)$ from each entrepreneur and monitors if $L(y, \theta) < \bar{L}$, incurring cost c_b. The bank's total revenue is given by $\pi = q \sum_{j=1}^{m} min(L(y), \bar{L})$. The monitoring cost is given by $C = c_b N(s)$, where $N(s)$ is the number of entrepreneurs that default. As $m \to \infty$, the bank diversifies idiosyncratic risk. By the law of large

[18] In a simple debt contract the lender receives the entire realization when bankruptcy occurs. In an arbitrary debt contracts the borrower may retain some output.

numbers, the expected revenue for a bank with a loan portfolio of size m is

$$p\lim_{m\to\infty}\frac{1}{mq}\pi = \int_{B_b} Ł(y,\theta)dG(y,\theta;s) + \int_{B_b'} \bar{L}dG(y,\theta;s) = \pi(L,\theta).$$

Monitoring cost c_b has a binomial distribution with parameters m and $p = \int_{B_b} dG(y,\theta;s)$. As $m \to \infty$ it follows that[19]

$$p\lim_{m\to\infty}\frac{1}{mq}C = \frac{c_b}{q}\int_{B_b} dG(y,\theta;s).$$

Thus when the bank never fails, i.e., $s = \bar{s}$, the expected return function for a bank that contracts with an infinite number of entrepreneurs is given by (1). Williamson showed that the depositors' expected cost of monitoring the bank goes to zero as portfolio size goes to infinity because the portfolio earns $\bar{L}$ with probability one. The bank can then pay depositors reservation value $\bar{D}$ with certainty and the cost of delegation is nil.

Finally, Williamson showed that the bank's expected return function is concave in loan rate $\bar{L}$, thus it has an interior maximum at some $\bar{L}^*$. This leads to equilibrium credit rationing by loan quantity at $\bar{L}^*$. Even if a rationed borrower offered to pay a loan rate higher than $\bar{L}^*$, the bank would refuse because $\bar{L}^*$ maximizes the bank's expected return. The intuition for this credit rationing by loan quantity is that when failure is costly to the lender, an increase in the loan rate may decrease the bank's expected return because it raises the probability of borrower default.

[19] Given that $N(s)$ is a binomial distribution, by the Law of Large Numbers it converges to $m.p$ and m cancels out. With no default risk the bank does not default in the limit.

Appendix 2

Equation (1), which must equal $\bar{D}$, and (4), which must be non-negative, specify the banks's expected profit requirements without and with default risk, respectively. The crucial difference is term $\sum\limits_{s=l,h} p_s D(\pi_b(L,\theta;s))$, which is the expected payment from the bank to depositors. Because a risky bank will sometimes default, depositors expect to incur monitoring costs. These expected monitoring costs raise the "effective reservation return" that depositors must receive. We now consider the implications of this.

When there is default risk, as $m \to \infty$ depositor incentive compatibility constraint (3) can be written

$$\sum_{s=l,h} p_s \left[D(\pi_b(L,\theta;s)) + qc_d M(s) \right] \geq \bar{D}. \tag{12}$$

This equation indicates that depositors must be compensated for expected monitoring costs. As a bank diversifies idiosyncratic risk it obtains $D(\pi_b(L,\theta;s))$ to compensate depositors. But with default risk, the deadweight monitoring cost must be accounted for. For some states $M(s) = 1$, and depositors incur monitoring cost qc_d. If the bank is not risky, then $M(s) = 0$ and (5) simplifies to $\sum\limits_{s=l,h} p_s \left[D(\pi_b(L,\theta,s)) \right] \geq \bar{D}$.

The key insight is that the bank cannot eliminate default risk, even with an infinite number of projects. Thus rewriting (4), depositors wish to insure that the bank's profit is high enough to enable them to recover their expected monitoring costs in bankruptcy states. That is,

$$\sum_{s=l,h} p_s \int\limits_{B_b} (L(y,\theta) - \frac{c_b}{q}) dG(\cdot) + \int\limits_{B_b'} \bar{L} dG(\cdot) \geq \sum_{s=l,h} p_s D(\pi_b(L,\theta,s)). \tag{13}$$

The left hand side is the bank's expected return with no default risk. When $M(s) = 0$, (6) reduces to (1). If the bank defaults in state $s = l$, $M(l) = 1$. Then $B_d = L(y,\theta) \quad D(\pi_b(L,\theta;l)) < \bar{D}$, and depositors monitor the bank with probability $p_l \geq 0$. Evaluating depositor incentive constraint (5) gives

$$\sum_{s=l,h} p_s D(\pi_b(L,\theta;s)) \geq \bar{D} + p_l q c_d. \tag{14}$$

Given (6), the depositor's incentive constraint can be written

$$\sum_{s=l,h} p_s \left[\int\limits_{B^b} (L(y,\theta) - \frac{c_b}{q}) dG(y,\theta;s) + \int\limits_{B_b'} \bar{L} dG(y,\theta;s) \right] \geq \bar{D} + p_l q c_d. \quad (15)$$

Or, the risky bank's expected return must be sufficiently high to compensate a depositor for both the opportunity cost of the reservation project and the expected cost of recovering funds from the risky bank when it defaults. Thus, (8) can be written

$$\Pi(L(y,\theta;s),\theta) \geq \bar{D} + p_l q c^d = \psi(\theta). \quad (16)$$

Proof of Proposition 1. Recall (1)

$$\Pi(L(y,\theta),\theta;s) = \int_{B^b} (L(y,\theta) - \frac{c_b}{q}) dG(y,\theta;s) + \int_{B_b'} \bar{L} dG(y,\theta;s).$$

Integrating by parts and solving gives

$$\Pi(L(y,\theta),\theta;s) = \bar{L} - \frac{c_b}{q} G(\bar{L},\theta;s) - \int\limits_0^{\bar{L}} dG(y,\theta;s).$$

Let $\bar{L} = x$. Part (a) shows that, as in Williamson, $\pi(x,\theta)$ reaches a maximum for x, given θ. Clearly

$$\pi'(x,\theta;s) = 1 - \frac{c_b}{q} g(x,\theta;s) - G(x,\theta;s) = 0.$$

Solving this equation gives $x^* = \eta(\theta)$; $\bar{L}^*(\theta) = \eta(\theta)$ is the optimal loan rate.

The assumption that $1 > \frac{c_b}{q} g(0,\theta;s)$, $\forall\theta$, assures that the profit function reaches an interior maximum for θ. Using the assumption

$$\lim_{x\to 0} \pi'(x,\theta;s) = 1 - \frac{c_b}{q} g(0,\theta;s) - G(0,\theta;s) \geq 0,$$

$$\lim_{x\to \bar{y}} \pi'(x,\theta;s) = 1 - \frac{c_b}{q} g(\bar{y},\theta;s) - 1 \leq 0.$$

Further,

$$\pi''(x,\theta;s) = -\frac{c_b}{q} g'(x,\theta;s) - g(x,\theta;s).$$

Then $\pi(x, \theta; s)$ reaches a maximum for x as a function of θ, given the assumptions.

To prove part (b), recall that $\bar{L} = \bar{L}^*(\theta)$ is the loan rate that maximizes the expected profit function. Then $\bar{L} = \bar{L}^*(\theta)$ is the value such that

$$\pi'(\bar{L}^*(\theta), \theta; s) = 1 - \frac{c_b}{q} g(\bar{L}^*(\theta), \theta; s) - G(\bar{L}^*(\theta), \theta; s) = 0.$$

For $\bar{L}^*(\theta)$ to be a maximum, $\pi''(\bar{L}^*(\theta), \theta; s)$ must be less than zero. This is assured by the assumption $\frac{c_b}{q} g_x(x, \theta; s) + g(x, \theta; s) > 0$. The derivative of $\bar{L}^*(\theta)$ with respect to θ can be calculated using the implicit function theorem

$$\frac{d\bar{L}^*(\theta)}{d\theta} = -\frac{\frac{c_b}{q} g_\theta(\bar{L}, \theta; s) + G_\theta(\bar{L}, \theta; s)}{\frac{c_b}{q} g_{\bar{L}}(\bar{L}, \theta; s) + g(\bar{L}, \theta; s)}.$$

The assumption that $\bar{L}^*(\theta)$ does not change as θ changes holds as long as $\frac{c_b}{q} g_\theta(\bar{L}, \theta; s) + G_\theta(\bar{L}, \theta; s) = 0$. This requires $G_\theta(\bar{L}, \theta; s) = -\frac{c_b}{q} g_\theta(\bar{L}, \theta; s)$. The stochastic dominance assumption implies $G_\theta(\bar{L}, \theta; s) \geq 0$, then $g_\theta(\bar{L}, \theta; s) \leq 0$.[20]

Proof of Proposition 2. It follows from differentiation of the expected return function, when $\bar{L} = \bar{L}^*$ and for a given $\bar{D}$, that there is a maximum threshold quality level θ^A.

Suppose that $\bar{L} = \bar{L}^*$, where $\bar{L}^*$ is the value that maximizes the bank's expected revenue for a given θ. Differentiate the expected revenue function with respect to θ, and observe that $G_\theta \geq 0$ by Second Order Stochastic Dominance. Then

$$\pi(\bar{L}^*, \theta; s) = \pi_L(\bar{L}^*, \theta; s) L'(\theta) + \pi_\theta(\bar{L}^*, \theta; s).$$

Since $\bar{L}^*$ maximizes $\Pi(y, \theta)$, the first term is zero. Therefore

$$\pi_\theta(\bar{L}^*, \theta; s) = -\frac{c_b}{q} G_\theta(\bar{L}, \theta; s) - \int_0^{\bar{L}} G_\theta(y, \theta; s) \leq 0.$$

[20]Numerical examples using mean preserving changes in the variance as a measure of the quality of the distribution of project returns, θ, can be constructed in which there is no change in $\bar{L}^*(\theta)$ as θ changes. Other parameters can generate changes in either direction. Jaffe and Stigtliz (1990) analyze a similar problem and note that as the expected return function shifts down, the optimal loan rate can increase, decrease or stay the same. If the success probability of a risky project is reduced by the same proportion as the reduction in the success probability of the safe project, then the optimal loan rate does not change. If the risky project's success probability is reduced by more than proportionally compared with the safe project, then the loan rate will increase.

Then, the expected revenue function decreases as portfolio quality decreases. For a given $\bar{D}$ banks choose a quality threshold θ^A such that the expected revenue for $\bar{L}^*$ equals the opportunity cost of funds given by $\bar{D}$.

When $\bar{L} < \bar{L}^*$, for a given $\bar{D}$ and a fixed quality threshold θ, the bank chooses an interest rate on loans $\bar{L}$ such that expected revenue equals the opportunity cost given by $\bar{D}$. Any attempt to increase revenue would induce more depositors to become banks, which would drive expected revenue down.